9/3/17

$X_1, \ldots, X_n$ ανεξ. $\mu, \alpha$ σαθ

$Y = \sum_{i}^{n} a_i x_i$ , $E(Y) = \sum_{i}^{n} \alpha E(x_i)$ , $Var(Y) = \sum_{i}^{n} \alpha_i^2 Var(x_i)$

$X \sim N(\mu, \sigma^2)$ , $m_X(t) = e^{\mu t + \frac{\sigma^2}{2} t^2}$ | $X \sim G(\alpha, \beta)$ , $m_X(t) = (1-\beta t)^{-\alpha}$

Στατιστική ν Διηγματική Κατανομής

A, B | $n = 100$ | $X \sim B(n=100, p)$
$p = P(A)$ | $x =$ αρ. των A

$T = T(x_1, \ldots, x_n)$ | $\bar{x}$
$S = S(x_1, \ldots, x_n)$ | $s^2$
$s'$

Θεώρημα 1: Έστω $x_1, \ldots, x_n$ δείγμα από πληθυσμό $X$, με μέση τιμή $\mu$ και διακύμανση $\sigma^2$
$[E(x_i) = \mu, Var(x_i) = \sigma^2]$ Τότε:

i) $E(\bar{x}) = \mu$

ii) $Var(\bar{x}) = \frac{\sigma^2}{n}$

iii) $E(S^2) = \sigma^2$

Θεώρημα: $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ , $S^2 = \frac{1}{n-1}\sum (x_i - \bar{x})^2$

(i) $E(\bar{x}) = E\left[\frac{1}{n}\sum_{1}^{n} x_i\right] = \frac{1}{n}\sum_{1}^{n} E(x_i) = \frac{1}{n}\sum_{1}^{n} \mu = \mu$

(ii) $Var(\bar{x}) = Var\left[\frac{1}{n}\sum_{1}^{n} x_i\right] = \frac{1}{n^2}\sum^{n} Var\, x_i = \frac{1}{n^2}\sum_{i=1}^{n} \sigma^2 = \frac{\sigma^2}{n}$

(iii) $E(S^2) = \frac{1}{n-1} E\left[\sum_{i=1}^{n} (x_i - \bar{x})^2\right] =$

$$S^2 = \frac{1}{n-1}\sum_{1}^{n}(x_i-\bar{x})^2 = \frac{1}{n-1}\sum^{n}[(x_i-\mu)-(\bar{x}-\mu)]^2$$

$$= \frac{1}{n-1}\sum_{1}^{n}[(x_i-\mu)^2+(\bar{x}-\mu)^2-2(\bar{x}-\mu)(x_i-\mu)]$$

$$= \frac{1}{n-1}\Big[\sum_{i=1}^{n}(x_i-\mu)^2-n(\bar{x}-\mu)^2-2(\bar{x}-\mu)\sum_{i=1}^{n}(x_i-\mu)\Big]$$

$$= \frac{1}{n-1}\Big[\sum_{i=1}^{n}(x_i-\mu)^2+n(\bar{x}-\mu)^2\underbrace{-2\underbrace{(\bar{x}-\mu)(n\bar{x}-n\mu)}_{2n(\bar{x}-\mu)^2}}_{-n(\bar{x}-\mu)^2}\Big]$$

$$= \frac{1}{n-1}\Big[\sum(x_i-\mu)^2-n(\bar{x}-\mu)^2\Big]$$

και $E(S^2) = \frac{1}{n-1}E\Big[\sum(x_i-\mu)^2-n(\bar{x}-\mu)^2\Big]$

$$= \frac{1}{n-1}\Big[\sum_{i=1}^{n}E(x_i-\mu)^2-n\cdot E(\bar{x}-\mu)^2\Big]$$

$$= \frac{1}{n-1}\Big[\sum_{i=1}^{n}\sigma^2-n\cdot\frac{\sigma^2}{n}\Big] = \frac{1}{n-1}[n\sigma^2-\sigma^2] = \underline{\underline{\sigma^3}}$$

Παράδειγμα

$X: 3, 4, 5$ | $\mu = E(X) = \frac{1}{3}(3+4+5) = 4$

$P_X = \frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ | $E(X^2) = \frac{1}{3}(3^2+4^2+5^2) = 50$

$\sigma^2 = E(X^2)-\mu^2 = \frac{50}{3}-4 = 2/3$

$n=2$, με επανάθεση

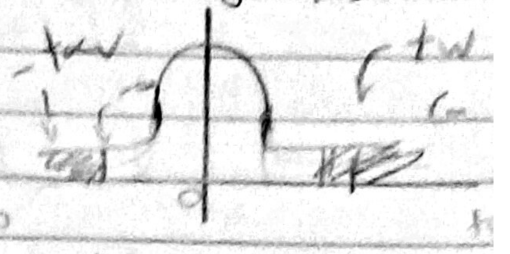
9 Δυνατά Δείγματα:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | (3,3) | (3,4) | (3,5) | (4,3) | (4,4) | (4,5) | (5,3) | (5,4) | (5,5) |
| $\bar{x}$: | 3 | 3,5 | 4 | 3,5 | 4 | 4,5 | 4 | 4,5 | 5 |
| $s^2$: | 0 | 0,5 | 2 | 0,5 | 0 | 0,5 | 2 | 0,5 | 0 |

Κατανομή $t_\nu$ - Τ με ν βαθμούς ελευθερίας

Αν $X \sim N(0,1)$ κ' $Y \sim X_\nu^2$ κ' X, Y ανεξάρτητες

$T = \frac{X}{\sqrt{Y/\nu}} \sim t_\nu$, $-\infty < t < +\infty$



$P(t_\nu > t_{\alpha,\nu}) = \alpha$

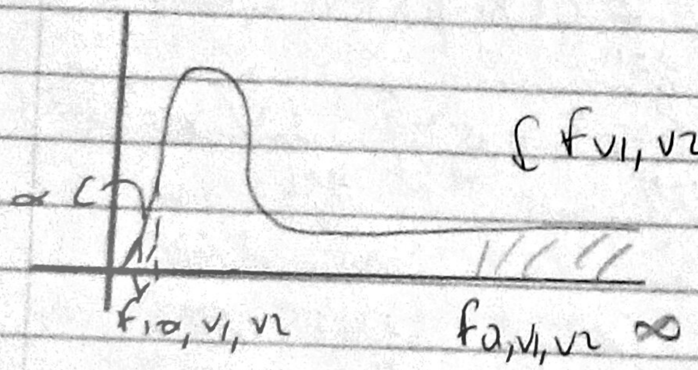
$$t_\nu^2 = \frac{X^2}{Y/\nu} = \frac{N(0,1)}{X_\nu^2/\nu} = \frac{X_1^2/1}{X_\nu^2/\nu} \sim F_{1,\nu}$$

Κατανομή $F_{\nu_1,\nu_2}$ - Ey

$\hookrightarrow$ $\nu_1$ κ' $\nu_2$ β.ε.

$F = \frac{X_{\nu_1}^2/\nu_1}{X_{\nu_2}^2/\nu_2} \sim F_{\nu_1,\nu_2}$ || $X_1 \sim X_{\nu_1}^2$ κ' $X_2 \sim X_{\nu_2}^2$

κ' $\frac{X_1/\nu_1}{X_2/\nu_2} \sim F_{\nu_1,\nu_2}$




Μπορεί κανείς να δει: $F_{1-\alpha,\nu_1,\nu_2} = \frac{1}{F(\alpha,\nu_2,\nu_1)}$

$t_\nu^2 \sim F_{1,\nu}$

$\bar{x}$ : 3   3,5   4   4.5   5

Κατανομή $\bar{x}$ : $p_{\bar{x}}$ : $\frac{1}{9}$   $\frac{2}{9}$   $\frac{3}{9}$   $\frac{2}{9}$   $\frac{1}{9}$

$E(\bar{x}) = \frac{1}{9}\cdot 3 + \ldots + \frac{1}{9}\cdot 5 = 4 \ (=\mu)$

$E(\bar{x}^2) = \frac{147}{9}$ , $\sigma^2_{\bar{x}} = \frac{147}{9} - 4^2 = \frac{3}{9} = \frac{1}{3} \ (= \frac{2/3}{2} = \frac{\sigma^2}{n})$

Κατανομή $S^2$ : $S^2$ : 0   0.5   2

$p_{S^2}$ : $\frac{3}{9}$   $\frac{4}{9}$   $\frac{2}{9}$

$E(S^2) = 0 * \frac{3}{9} + 0.5 * \frac{4}{9} + 2 * \frac{2}{9} = \frac{2}{3} \ (=\sigma^2)$

Θα μας χρειαστεί: $(n-1)S^2 = \sum_i (x_i-\mu)^2 - n(\bar{x}-\mu)^2$

$\chi_\nu^2$ – χι τετράγωνο με ν βαθμούς ελευθερίας

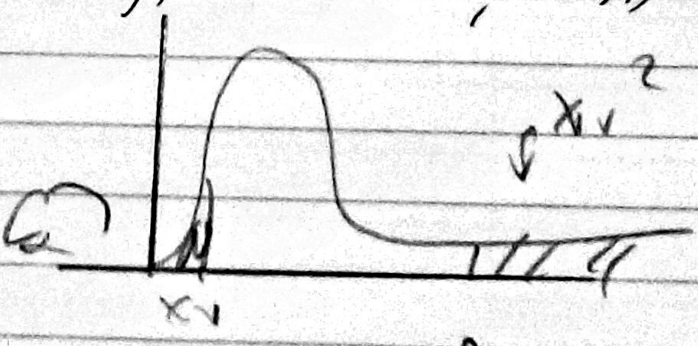
$\chi^2_\nu = G(\alpha = \frac{\nu}{2}, b=2)$ , $E(\chi^2_\nu) = \nu$ , $Var(\chi_\nu^2) = 2\nu$

Αν $Z_1, Z_2, \ldots, Z_\nu$ είναι $N(0,1)$ ανεξ. τ.μ.

$X = \sum_{i=1}^{\nu} Z_i^2$, τότε $X \sim \chi_\nu^2$. Συμβολικά: $\chi^2_\nu = \sum^{\nu} N^2(0,1)$

$X = \chi^2_{\nu_1} + \chi^2_{\nu_2} \sim \chi^2_{\nu_1+\nu_2}$

$m_X(t) = (1-2t)^{-\frac{\nu}{2}}$




$P(\chi_\nu^2 \geq \chi^2_{\nu,\alpha}) = \alpha$